Meni psiurei (1.4.) (N)

Диференціювання сложених функцій

(1.) $g(x,y) = f(\overbrace{2x^3+3y^2}^{u})$

$y\frac{\partial g}{\partial x} = x^2\frac{\partial g}{\partial y}$

$g_x = \frac{\partial g}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = f_u \cdot u_x$

$g_y = \frac{\partial g}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} = f_u \cdot u_y$

$u_x = 6x^2$

$u_y = 6y$

$g_x = f_u \cdot 6x^2$

$g_y = f_u \cdot 6y$ $\Big\} \Rightarrow y \cdot 6x^2 f_u = 6y \cdot x^2 f_u$ ✓

(2.)

$z = f(u, v)$

$u = x+y$ $\quad u_x = 1$ $\quad u_y = 1$

$v = x-y$ $\quad v_x = 1$ $\quad v_y = -1$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial u^2} - \frac{\partial^2 z}{\partial v^2} \Rightarrow z_{xy} = z_{uu} - z_{vv}$

$z_x = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = f_u u_x + f_v v_x = f_u + f_v$

$z_y = f_u u_y + f_v \cdot v_y = f_u - f_v$

$z_{xy} = (z_x)'_y =$ ~~$f_{uu} \cdot u_y + f_{uv} u_y - f_{uu} - f_{vv}$~~

$= (f_u + f_v)'_y = (f_u)'_y + (f_v)'_y$

$(f_u)'_y = \frac{\partial^2 f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial^2 f}{\partial u \partial v}\cdot\frac{\partial v}{\partial y} = f_{uu} u_y + f_{uv} \cdot v_y = f_{uu} - f_{uv}$

$(f_v)'_y = \frac{\partial^2 f}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial^2 f}{\partial v \partial u}\cdot\frac{\partial u}{\partial y} = f_{vv} v_y + f_{vu} \cdot u_y = f_{vu} - f_{vv}$

$z_{xy} = f_{uu}$ ~~$- f_{uv} + f_{vu}$~~ $- f_{vv} = f_{uu} - f_{vv}$ ✓ T

③ $z = z(x,y)$ Nemoguće CAA HEŽY! GAO!

$F(\underbrace{x+z}_{u}, \underbrace{y+z}_{v}) = 0$

$z_{xx} = ?$

$$\left.\begin{array}{l} u = x+z \\ v = y+z \end{array}\right\} z = z(x,y)$$

$\Rightarrow u_x = 1 + z_x$

$v_x = z_x$

$F_x = F_u u_x + F_v v_x = 0$ → F je bilo =0, ustoj je uvijek je nula

$F_u \cdot (1+z_x) + F_v \cdot z_x = F_u + F_u z_x + F_v z_x = F_u + z_x(F_u + F_v) = 0$

$$\Rightarrow \boxed{z_x = -\frac{F_u}{F_u + F_v}}$$

$F_{xx} = (F_u(1+z_x) + F_v z_x)'_x = 0$

$\Rightarrow (F_u)'_x (1+z_x) + F_u (1+z_x)'_x + (F_v)'_x z_x + F_v \cdot (z_x)'_x = 0$

$\Rightarrow (F_{uu} \cdot u_x + F_{uv} \cdot v_x)(1+z_x) + F_u \cdot z_{xx} + (F_{vv} v_x + F_{vu} u_x) z_x + F_v \cdot z_{xx} = 0$

$F_u \cdot z_{xx} + F_v \cdot z_{xx} = -(F_{uu} u_x + F_{uv} v_x)(1+z_x) - (F_{vv} v_x + F_{vu} u_x) z_x$

$z_{xx}(F_u + F_v) = -F_{uu} u_x - F_{uv} v_x - z_x(F_{uu} u_x + F_{uv} v_x) - z_x(F_{vv} v_x + F_{vu} u_x)$

$$z_{xx} = \frac{-F_{uu} u_x - F_{uv} v_x - z_x(F_{uu} u_x + F_{uv}(v_x + u_x) + F_{vv} v_x)}{F_u + F_v}$$

$$z_{xx} = \frac{-F_{uu} u_x - F_{vv} v_x + \frac{F_u}{F_u + F_v}(F_{uu} u_x + F_{uv}(u_x + v_x) + F_{vv} v_x)}{F_u + F_v}$$

$$z_{xx} = \frac{(-F_{uu} u_x - F_{vv} v_x)(F_u + F_v) + F_u(F_{uu} u_x + F_{uv}(u_x + v_x) + F_{vv} v_x)}{(F_u + F_v)^2}$$

$$= \frac{F_u(-F_{uu} u_x - F_{vv} v_x + F_{uu} u_x + F_{uv}(u_x + v_x) + F_{vv} v_x) + F_v \cdot F_{uu} u_x - F_v \cdot F_{vv} v_x}{(F_u + F_v)^2}$$

$$= \frac{F_u \cdot F_{uv}(u_x - v_x) - F_v F_{uu} u_x - F_v F_{vv} v_x}{(F_u + F_v)^2}$$

$$= \frac{F_u \cdot F_{uv}(1 + z_x + z_x) - F_v F_{uu}(1 + z_x) - F_v F_{vv} z_x}{(F_u + F_v)^2} =$$

$$z_{xx} = \frac{F_u \cdot F_{uu} + 2F_uF_{uv}z_x - F_vF_{uu} - F_vF_{uv}z_x - F_vF_{vv}z_x}{(F_u+F_v)^2} =$$

$$= \frac{F_u \cdot F_{uu} - F_vF_{uu} + z_x(2F_uF_{uv} - F_vF_{uu} - F_vF_{vv})}{(F_u+F_v)^2} =$$

$$= \frac{F_u + F_{uv} - F_vF_{uu} + \frac{F_u}{F_u+F_v}(2F_uF_{uv} - F_vF_{uu} + F_vF_{vv})}{(F_u+F_v)^2} =$$

$$= \frac{(F_u+F_v)(-F_uF_{uv} - F_vF_{uu}) - F_u(2F_uF_{uv} - F_vF_{uu} - F_vF_{vv})}{(F_u+F_v)^3} =$$

$$= \frac{-F_u^2F_{uv} - F_u \cdot F_v + F_{uu}(F_uF_v F_{uv} - F_v^2F_{uu} + 2F_u^2 \cdot F_{uv} + F_uF_vF_{uu} + F_uF_vF_{vv})}{(F_u+F_v)^3} =$$

$$= \frac{-3F_u^2F_{uv} + F_uF_v(F_{vv} - F_{uv}) - F_v^2F_{uu}}{(F_u+F_v)^3}$$

$z_{xx} = xz^2 + ye^z + z \qquad z_x = ?$

$z_x = z^2 + x \cdot 2z \cdot z_x + y \cdot e^z \cdot z_x$

$z^2 = 2zz_x + ye^z z_x - z_x = 0$

$-z_x(ye^z - 2x - 1) = z^2$

$$\boxed{z_x = \frac{z^2}{2x - ye^z + 1}}$$

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(3) $z = z(x,y)$

$F(x+z, y+z) = 0$

$z_{xx} = ?$

$u = x+z \qquad u_x = 1 + z_x$

$v = y+z \qquad v_x = z_x$

$F_x = 0$

$F_x = F_u u_x + F_v v_x = F_u(1+z_x) + F_v(z_x) = F_u + F_u \cdot z_x + F_v z_x =$

$= F_u + z_x(F_u + F_v) = 0$

$z_x(F_u+F_v) = -F_u$

$$\boxed{z_x = -\frac{F_u}{F_u+F_v}} \qquad u_x = 1 + z_x = \frac{F_u + F_v - F_u}{F_u+F_v} = \frac{F_v}{F_u+F_v}$$

$$\boxed{u_x = \frac{F_v}{F_u+F_v}} \qquad \boxed{v_x = -\frac{F_u}{F_u+F_v}}$$

$F_{xx} = 0$

$F_{xx} = (F_x)'_x = (F_u u_x + F_v v_x)_x = (F_u(1+z_x) + F_v z_x)'_x =$

$= (F_u)'_x (1+z_x) + F_u (1+z_x)'_x + (F_v)'_x z_x + F_v z_{xx} =$

$= (F_{uu} u_x + F_{uv} v_x)(1+z_x) + F_u z_{xx} + (F_{vv} v_x + F_{vu} u_x) z_x + F_v z_{xx} =$

$= z_{xx}(F_u + F_v) + F_{uu} u_x + F_{uv} v_x + z_x (F_{uu} u_x + F_{uv}(v_x + u_x) + F_{vv} v_x) =$

$=$

$z_{xx}(F_u + F_v) = -F_{uu} \cdot \frac{F_v}{F_u + F_v} + F_{uv} \cdot \frac{F_u}{F_u + F_v} + \frac{F_u}{F_u + F_v} \left(F_{uu} \cdot \frac{F_v}{F_u + F_v} + F_{uv} \cdot \frac{F_v - F_u}{F_v + F_u} - F_{vv} \cdot \frac{F_u}{F_u + F_v} \right)$

$z_{xx}(F_u + F_v) = \frac{F_{uv} \cdot F_u - F_{uu} F_v}{F_u + F_v} + \frac{F_u (F_{uu} F_v + F_{uv}(F_v - F_u) - F_{vv} \cdot F_u)}{(F_u + F_v)^2} =$

$= \frac{(F_u + F_v)(F_{uv} F_u - F_{uu} F_v) + F_u (F_{uu} F_v + F_{uv}(F_v - F_u) - F_{vv} F_u)}{(F_u + F_v)^2}$

$= \frac{F_u (F_{uv} F_u - F_{uu} F_v + F_{uu} F_v + F_{uv}(F_v - F_u) - F_{vv} F_u) - F_v^2 F_{uu}}{(F_u + F_v)^2}$

$= \frac{F_u^2 F_{uv} + F_{uv} F_v - F_{uv} F_u - F_v^2 F_{uu}}{(F_u + F_v)^2}$

БАБО!

$(F_{vu} u_x + F_{vv} v_x)(1+z_x) + F_v z_{xx} + (F_{vu} u_x + F_{vv} v_x) z_x + F_v z_{xx} = 0$

$(F_{vu}(1+z_x) + F_{vv} z_x)(1+z_x) + (F_{vu}(1+z_x) + F_{vv} z_x) z_x + z_{xx}(F_u + F_v) = 0$

$(F_{vu} + F_{vu} z_x + F_{vv} z_x)(1+z_x) + (F_{vu} + F_{vu} z_x + F_{vv} z_x) z_x + z_{xx}(F_u + F_v) = 0$

$F_{vu} + F_{vu} z_x + F_{vv} z_x + F_{vu} z_x + F_{vu} z_x^2 + F_{vv} z_x^2 + F_{vu} z_x + F_{vu} z_x^2 + F_{vv} z_x^2 + z_{xx}(F_u + F_v) = 0$

$F_{vu} + 2F_{vu} z_x + 2F_{vv} z_x + F_{vu} z_x^2 + 2F_{vv} z_x^2 + F_{vv} z_x^2 + z_{xx}(F_u + F_v) = 0$

$F_{vu} - \frac{2F_{vu} F_v}{F_u + F_v} - \frac{2F_{vv} F_u}{F_u + F_v} + F_{vu} \cdot \frac{F_u^2}{(F_u + F_v)^2} + 2F_{vv} \cdot \frac{F_u^2}{(F_u + F_v)^2} + F_{vv} \cdot \frac{F_u^2}{(F_u + F_v)^2} + z_{xx}(F_u + F_v) = 0$

$F_{vu}(F_u + F_v)^2 - 2F_{vu} F_v (F_u + F_v) + 2F_{vv} F_u (F_u + F_v) + F_{vu} F_u^2 + 2F_{vv} F_u^2 + F_{vv} F_u^2 + z_{xx}(F_u + F_v) = 0$

$F_{vu} F_u^2 + 2F_{vu} F_v F_u - 2F_{vu} F_v^2 + 2F_{vu} F_u F_v + 2F_{vv} F_u^2 - 2F_{vv} F_u F_v + F_{vu} F_u^2 + 2F_{vv} F_u^2 + F_{vv} F_u^2 = -z_{xx}(F_u + F_v)$

$(F_u + F_v)^2$

$-2F_{vu} F_u F_v + F_{vv} F_u^2 = -z_{xx}(F_u + F_v)$

$(F_u + F_v)^2$

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(3.) $z_x = -\frac{F_U}{F_U + F_M}$

$U_x = 1 + z_x$

$M_x = z_x$

$F_x = F_U(1+z_x) + F_M \cdot z_x$

$F_{xx} = (F_{UU}U_x + F_{UM}M_x)(1+z_x) + F_U z_{xx} + (F_{MM}M_x + F_{MU}U_x)z_x + F_M z_{xx} = 0$

$\Rightarrow (F_{UU}(1+z_x) + F_{UM}\cdot z_x)(1+z_x) + F_U z_{xx} + (F_{MM}z_x + F_{MU}(1+z_x))z_x + F_M z_{xx} =$

$= (F_{UU} + F_{UU}z_x + F_{UM}z_x)(1+z_x) + (F_{MM}z_x + F_{MU} + F_{MU}z_x)z_x + z_{xx}(F_U + F_M) =$

$= F_{UU} + F_{UU}z_x + F_{UM}z_x + F_{UU}z_x + F_{UU}z_x^2 + F_{UM}z_x^2 + F_{MM}z_x^2 + F_{MU}z_x + F_{UM}z_x^2 + z_{xx}(F_U + F_M) =$

$= F_{UU} + 2F_{UU}z_x + 2F_{UM}z_x + F_{UU}z_x^2 + 2F_{UM}z_x^2 + F_{MM}z_x^2 + z_{xx}(F_U + F_M) =$

$= F_{UU} - \frac{2F_{UU}F_U}{F_U+F_M} - \frac{2F_{UM}F_U}{F_U+F_M} + \frac{F_{UU}F_U^2}{(F_U+F_M)^2} + \frac{2F_{UM}F_U^2}{(F_U+F_M)^2} + \frac{F_{MM}F_U^2}{(F_U+F_M)^2} + z_{xx}(F_U+F_M) =$

$= \frac{F_{UU}(F_U+F_M)^2 - 2F_{UU}F_U(F_U+F_M) - 2F_{UM}F_U(F_U+F_M) + F_{UU}F_U^2 + 2F_{UM}F_U^2 + F_{MM}F_U^2}{(F_U+F_M)^2} + z_{xx}(F_U+F_M) =$

$= \frac{F_{UU}F_U^2 + 2F_{UU}F_UF_M + F_{UU}F_M^2 - 2F_{UU}F_U^2 - 2F_{UU}F_UF_M - 2F_{UM}F_U^2 - 2F_{UM}F_UF_M + F_{UU}F_U^2 + 2F_{UM}F_U^2 + F_{MM}F_U^2}{(F_U+F_M)^2} + z_{xx}(F_U+F_M) =$

$= \frac{F_{UU}F_M^2 - 2F_{UM}F_UF_M + F_{MM}F_U^2}{(F_U+F_M)^2} + z_{xx}(F_U+F_M) =$

$\Rightarrow z_{xx}(F_U+F_M) = -\frac{F_{UU}F_M^2 + 2F_{UM}F_UF_M - F_{MM}F_U^2}{(F_U+F_M)^2}$

$$\boxed{z_{xx} = -\frac{F_{UU}F_U^2 + 2F_{UU}F_UF_U - F_{UU}F_U^2}{(F_U+F_U)^3}}$$

(4.) $z = \sqrt{\frac{x}{y}}f(xy) + g\left(\frac{x}{y}\right)$ $\quad x^2 z_{xx} - y^2 z_{yy} + 2yz_y = ?$

$u = \frac{x}{y}$, $M = xy$ $\rightarrow$ $z = \sqrt{u}f(M) + g(u)$

$$\boxed{\begin{array}{ll} U_x = \frac{1}{y} & M_x = y \\ U_y = -\frac{x}{y^2} & M_y = x \end{array}}$$

$$z_x = z_U U_x + z_M M_x = z_U \cdot \frac{1}{y} + z_M \cdot y$$

$$z_{xx} = (z_{UU} U_x + z_{UM} M_x)\frac{1}{y} + (z_{MM} M_x + z_{MU} U_x) y =$$

$$= \left(z_{UU} \cdot \frac{1}{y} + z_{UM} \cdot y\right)\frac{1}{y} + \left(z_{MM} y + z_{MU} \cdot \frac{1}{y}\right) y =$$

$$= z_{UU} \cdot \frac{1}{y^2} + z_{UM} + z_{MM} y^2 + z_{MU} =$$

$$= \boxed{z_{UU} \frac{1}{y^2} + 2 z_{UM} + z_{MM} y^2}$$

$$z_y = z_U U_y + z_M M_y = \boxed{z_U \cdot \left(-\frac{x}{y^2}\right) + z_M \cdot x}$$

$$z_{yy} = \left(z_{UU} U_y + z_{UM} M_y\right)\left(-\frac{x}{y^2}\right) + z_U \cdot \frac{2x}{y^3} + \left(z_{MM} M_y + z_{MU} U_y\right) x =$$

$$= \left(z_{UU} \cdot \left(-\frac{x}{y^2}\right) + z_{UM} \cdot x\right)\left(-\frac{x}{y^2}\right) + z_U \cdot \frac{2x}{y^3} + \left(z_{MM} \cdot x + z_{MU} \cdot \left(-\frac{x}{y^2}\right)\right) x =$$

$$= z_{UU} \cdot \frac{x^2}{y^4} - z_{UM} \frac{x^2}{y^2} + z_U \frac{2x}{y^3} + z_{MM} x^2 - z_{MU} \frac{x^2}{y^2} =$$

$$= \boxed{z_{UU} \cdot \frac{x^2}{y^4} - 2 z_{UM} \frac{x^2}{y^2} + z_U \frac{2x}{y^3} + z_{MM} \cdot x^2}$$

$$x^2 \cdot z_{xx} - y^2 z_{yy} - z_y z_y = A$$

$$A = x^2\left(z_{UU} \frac{1}{y^2} + 2 z_{UM} + z_{MM} \cdot y^2\right) - y^2\left(z_{UU} \frac{x^2}{y^4} - 2 z_{UM} \frac{x^2}{y^2} + z_U \frac{2x}{y^3} + z_{MM} x^2\right) -$$

$$+ 2y\left(+ z_U \cdot \frac{x}{y^2} + z_M \cdot x\right)$$

$$A = \left(z_{UU} \frac{x^2}{y^2}\right) + \left(z_{UM} \cdot 2x^2\right) + z_{MM} \cdot x^2 y^2 - \left(z_{UU} \cdot \frac{x^2}{y^2}\right) + \left(z_{UM} 2x^2\right) - z_U \cdot \frac{2x}{y} - z_{MM} x^2 y^2 + z_U \frac{2x}{y} + z_M \cdot 2xy$$

$$\boxed{A = z_{UM} \cdot 4x^2 + z_M \cdot 2xy}$$

(1.) $x \cdot z_x + y z_y = ?$

$z(x, y) = z$

$F\left(\frac{x}{z}, \frac{1}{z}\right) = 0$

$$u = \frac{x}{z} \qquad M = \frac{y}{z}$$

$$\boxed{u_x = \frac{z - x \cdot z_x}{z^2}} \qquad \boxed{M_x = -\frac{M}{z^2} \cdot z_x}$$

$$\boxed{u_y = -\frac{x}{z^2} \cdot z_y} \qquad \boxed{M_y = \frac{z - M z_M}{z^2}}$$

$$F_x = F_{UU} U_x + F_{MM} M_x = F_{UU} \cdot \frac{z - x z_x}{z^2} + F_{MM} \cdot \left(-\frac{y \cdot z_x}{z^2}\right) =$$

$$= \frac{F_{UU} \cdot z - F_{UU} \cdot x z_x - F_{MM} y z_x}{z^2} = 0$$

$$F_{UU} z = z_x\left(F_{UU} x + F_{MM} y\right)$$

$$\boxed{z_x = \frac{F_{UU} \cdot z}{F_{UU} \cdot x + F_{MM} \cdot y}}$$

$$F_y = F_U U_y + F_M M_y = F_U \cdot \left(-\frac{x \cdot z_y}{z^2}\right) + F_M \cdot \frac{z - y \cdot z_y}{z^2} = 0$$

$$\Rightarrow -F_U \cdot x \cdot z_y + F_M \cdot z - F_M \cdot y \cdot z_y = 0$$

$$F_M z = z_y (F_U \cdot x + F_M y)$$

$$\boxed{z_y = \frac{F_M z}{F_U \cdot x + F_M y}}$$

$$A = x \cdot z_x + y \cdot z_y$$

$$A = x \cdot \frac{F_U \cdot z}{x \cdot F_U + y \cdot F_M} + y \cdot \frac{F_M \cdot z}{F_U \cdot x + F_M \cdot y} =$$

$$A = \frac{F_U \cdot x \cdot z + F_M \cdot y \cdot z}{x F_U + y F_M} = \frac{z (F_U x + F_M \cdot y)}{F_U x + F_M y} = z$$

$$\boxed{A = z}$$

(2.) $z = z(x, y)$

$$F\left(\frac{1}{x+y} + \frac{1}{z}, \frac{1}{x-y} + \frac{1}{z}\right) = 0$$

$$(x^2 + y^2) z_x + 2xy\, z_y + z^2 = ?$$

$$U = \frac{1}{x+y} + \frac{1}{z}$$

$$\boxed{U_x = -\frac{1}{(x+y)^2} - \frac{z_x}{z^2}}$$

$$\boxed{U_y = -\frac{1}{(x+y)^2} - \frac{z_y}{z^2}}$$

$$M = \frac{1}{x-y} + \frac{1}{z}$$

$$\boxed{M_x = -\frac{1}{(x-y)^2} - \frac{z_x}{z^2}}$$

$$\boxed{M_y = \frac{1}{(x-y)^2} - \frac{z_y}{z^2}}$$

$$F_x = F_U U_x + F_M M_x = F_U \cdot \left(-\frac{1}{(x+y)^2} - \frac{z_x}{z^2}\right) + F_M \cdot \left(-\frac{1}{(x-y)^2} - \frac{z_x}{z^2}\right) = 0$$

(4.)

$$-\frac{F_U}{(x+y)^2} - \frac{F_U \cdot z_x}{z^2} - \frac{F_M}{(x-y)^2} - \frac{F_M \cdot z_x}{z^2} = 0$$

$$-\frac{F_U z_x + F_M z_x}{z^2} = \frac{F_U}{(x+y)^2} + \frac{F_M}{(x-y)^2}$$

$$U = \frac{y}{x}$$

$$\boxed{U_x = -\frac{y}{x^2}}$$

$$\boxed{U_y = -\frac{x}{y^2}}$$

$$\frac{z_x (F_U + F_M)}{z^2} = -\left(\frac{F_U (x-y)^2 + F_M (x+y)^2}{(x+y)^2 (x-y)^2}\right)$$

$$\boxed{z_x = -\frac{z^2 \left(F_U (x-y)^2 + F_M (x+y)^2\right)}{(F_U + F_M)(x+y)^2 (x-y)^2}}$$

$$F_y = F_U U_y + F_M M_y = F_U \cdot \left(-\frac{1}{(x+y)^2} - \frac{z_y}{z^2}\right) + F_M\left(\frac{1}{(x-y)^2} - \frac{z_y}{z^2}\right) = 0$$

$$-\frac{F_U}{(x+y)^2} - \frac{F_U \cdot z_y}{z^2} + \frac{F_M}{(x-y)^2} - \frac{F_M \cdot z_y}{z^2} = 0$$

$$\frac{F_U z_y + F_M z_y}{z^2} = \frac{F_M}{(x-y)^2} - \frac{F_U}{(x+y)^2}$$

$$\frac{z_y(F_U+F_M)}{z^2} = \frac{F_M(x+y)^2 + F_U(x-y)^2}{(x+y)^2(x-y)^2}$$

$$\boxed{z_y = \frac{z^2\left(F_M(x+y)^2 - F_U(x-y)^2\right)}{(F_U+F_M)(x+y)^2(x-y)^2}}$$

$$A = (x^2+y^2)z_x + 2xy\, z_y + z^2$$

$$A = (x^2+y^2)\left(-\frac{z^2\left(F_U(x-y)^2 + F_M(x+y)^2\right)}{(F_U+F_M)(x^2-y^2)^2}\right) + 2xy \cdot \frac{z^2\left(F_M(x+y)^2 - F_U(x-y)^2\right)}{(F_U+F_M)(x^2-y^2)^2} + z^2$$

$$A = \frac{-z^2(x^2+y^2)\left(F_U(x-y)^2 + F_M(x+y)^2\right) + z^2 2xy\left(F_M(x+y)^2 - F_U(x-y)^2\right) + z^2\left((F_U+F_M)(x^2-y^2)^2\right)}{(F_U+F_M)(x^2-y^2)^2}$$

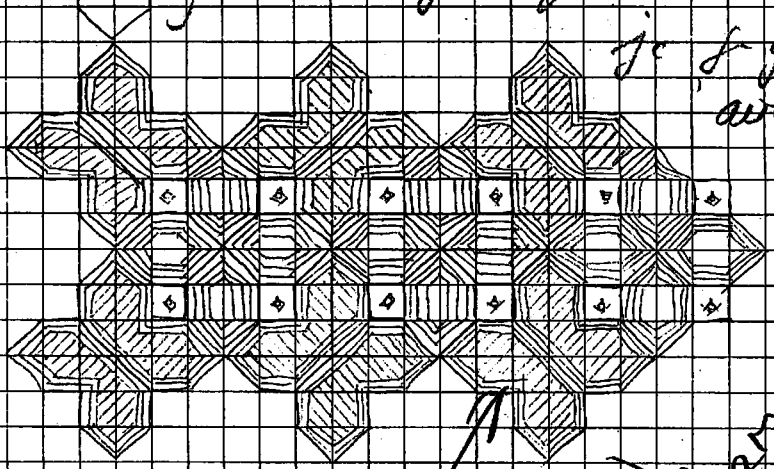
$$A = \frac{-F_U(x-y)^2 \cdot z^2(x^2+y^2) + F_M(x+y)^2 \cdot z^2(x^2+y^2) + F_M(x+y)^2 z^2 \cdot 2xy - F_U(x-y)^2 z^2 \cdot 2xy + z^2\left((F_U+F_M)(x^2-y^2)^2\right)}{(F_U+F_M)(x^2-y^2)^2}$$

$$A = \frac{-F_U(x-y)^2\left(z^2(x^2+y^2) + z^2 2xy\right) + F_M(x+y)^2\left(-z^2(x^2+y^2) + z^2 2xy\right) + z^2\left((F_U+F_M)(x^2-y^2)^2\right)}{(F_U+F_M)(x^2-y^2)^2}$$

$$A = \frac{-F_U(x-y)^2 \cdot z^2(x+y)^2 - F_M(x+y)^2 \cdot z^2(x-y)^2 + z^2\left((F_U+F_M)(x^2-y^2)^2\right)}{(F_U+F_M)(x^2-y^2)^2}$$

$$A = \frac{-z^2\left(F_U(x-y)^2 + F_M(x+y)^2\right) + z^2\left((F_U+F_M)(x^2-y^2)^2\right)}{(F_U+F_M)(x^2-y^2)^2}$$

$$A = \frac{(F_U+F_M)(x^2-y^2)^2(-z^2+z^2)}{(F_U+F_M)(x^2-y^2)^2} = 0$$



$$\frac{1}{y^2} = \frac{-1 \cdot 2y}{y^4} = -\frac{2}{y^3}$$

$$\frac{e^x}{(x+2)^2} = -e^x \cdot \left(\frac{1}{(x+2)^2}\right)' = -e^x \cdot \left(-\frac{2}{(x+2)^3}\right) \cdot 2(x+2) \cdot 1$$